THERMAL INTERACTION BETWEEN WATER FLOW AND FROZEN GROUND UNDER HYDROEROSION

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A model for one of the methods of thermal reclamation of frozen ground is proposed which allows for water flow temperature variations on the ground surface due to heat exchange between the water flow and a thawing solid. In a computation experiment, the specific features of phase transition front propagation and liquid temperature variation are studied.

At present, one of the methods to prepare frozen rocks for their development is their thawing by different heat sources [1]. The set of processes that occur in such a situation is called water-thermal reclamation of the frozen rocks, which is aimed at goal-directed variations of their strength and other properties by controlling the temperature, phase composition of the pore solution, and freezing ability of rock-forming components. Hydraulic thawing due to water flow supplied to the frozen rock surface is a widespread mode of water-thermal reclamation.

A mathematical model for heat exchange between the liquid flow and the solid (semispace) with no phase transition and constant thermophysical parameters is considered in [2]. There, the analytical solution to the problem corresponding to a constant inlet water flow temperature is obtained, too.

In [3], to describe this process a model for solid (frozen rock) ablation at an assigned heat flux on its surface is adopted. The corresponding nonlinear boundary-value problem is solved by the method where the front is flattened by the explicit finite-difference scheme. Such a statement sets a limit on the possibilities of analyzing this process because it does not allow for liquid temperature variation as the liquid spreads over the frozen rock surface.

In the present article, the results from [3] are extended to the thermal interaction of the water flow with the frozen ground. For the mathematical statement of the problem the following simplifying assumptions are made: the rock mass in the thawing and frozen states is uniform and isotropic; the thermophysical properties of the rock and the liquid are constant; the hydraulic flow characteristics (depth and velocity) are constant. Then the mathematical model of the process may be given in the form of the following system of differential equations:

$$\frac{\partial \Theta_1}{\partial \tau} = \varkappa \frac{\partial^2 \Theta_1}{\partial z^2}, \quad 0 < z < s(x,\tau), \ 0 \leqslant x \leqslant L, \ \tau > 0; \tag{1}$$

$$\frac{\partial \Theta_2}{\partial \tau} = \frac{\partial^2 \Theta_2}{\partial z^2}, \quad z < s(x,\tau); \tag{2}$$

$$\frac{\partial \Theta_l}{\partial x} + \beta(\Theta_l - \Theta_1) = 0, \quad 0 < x \leqslant L, \ z = 0; \tag{3}$$

$$\Theta_l = 1, \ z = 0, \ x = 0; \tag{4}$$

$$\frac{\partial \Theta_1}{\partial z} = \alpha(\Theta_1 - \Theta_l), \ z = 0, \ 0 \leqslant x \leqslant L; \tag{5}$$

$$\Theta_1 = \Theta_2 = \Theta_{ph}, \ z = s(x,\tau); \tag{6}$$

$$-\lambda_1 \frac{\partial \Theta_1}{\partial z} + \lambda_2 \frac{\partial \Theta_2}{\partial z} = \frac{ds}{d\tau}, \ z = s(x,\tau); \tag{7}$$

$$\lim_{z\to\infty} \Theta_2 = 0;$$

$$\Theta_2 = 0, \ s = 0, \ \tau = 0, \tag{9}$$

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where

$$\tau = a_2 t/h^2;\ x = \bar{x}/h;\ z = \bar{z}/h;\ s(x,\ \tau) = \bar{s}(\bar{x},\ t)/h;$$

$$\Theta_1 = \frac{T_1 - T_0}{T_{in} - T};\ \Theta_2 = \frac{T_2 - T_0}{T_{in} - T_0};\ \Theta_l = \frac{T_l - T_0}{T_{in} - T_0};\ \Theta_{ph} = \frac{T_{ph} - T_0}{T_{in} - T_0};$$

$$\varkappa = \frac{a_1}{a_2};\ \alpha = \frac{h\bar{\alpha}}{\bar{\lambda}_1};\ \beta = \frac{\bar{\alpha}}{c_l \rho_l V};\ \lambda_i = \frac{\bar{\lambda}_i (T_{in} - T_0)}{l\rho w a_2},\ i = \overline{1,\ 2}.$$

In system (1)-(9), equations (1)-(2) under boundary and initial conditions (5)-(6) describe the thawing of the water-saturated ground, and equation (3) under boundary condition (4), the liquid flow temperature variation on the ground surface. In writing equation (3), the obvious thawing assumption has been used: the transient process rate in the liquid is much greater than the phase transition front propagation velocity in the ground. In addition, it has also been assumed that the amount of liquid which is exchanged between the water flow and the ground due to filtration in the thawed layer is small, as compared to the volume water flow rate. The possibility of using a quasi-one-dimensional model is substantiated in [3].

Boundary condition (5) and equation (3) contain the assumption that the thawed layer is formed from the very start of the process. In reality, because of the thermal resistance, some time is needed for this, which can be found by solving the appropriate conjugate heat transfer problem with no phase transition: equation (2); equation (3) and boundary condition (5) where θ_1 is replaced by θ_2, and the thermal conductivity of the frozen ground is used to determine α. Use of the time-integral Laplace transformation yields the following transcendental equation:

$$1 - \Theta_{ph} = \exp(y^2)\,\mathrm{erfc}(y), \tag{10}$$

where $y = \alpha[\rho \exp(\beta x)]^{1/2}$.

For the values of the parameters at which the computation experiment has been made, the maximum value of the dimensionless time before the thawing starts is 0.2, which corresponds to about 1 min of real time.

To solve Stefan's problem (1), (2), (5)-(9) use is made of the algorithm of "catching" the phase transition front at the spatial grid node, which provides the highest accuracy in solving Stefan's one-dimensional problems with monotonic interface motion [4, 5]. According to the method, let us introduce a grid with the constant step Δz with respect to the spatial variable z. The time step $\Delta\tau j$ is determined so that the end of the broken line approximating the curve $z = s(\tau)$ would move along the z coordinate by the amount Δz. To solve the corresponding system of algebraic equations an elimination algorithm is used.

The solution to conjugate problem (1)-(9) has been obtained by the following computation algorithm, similar to splitting into the physical processes.

1. The initial approximation of the function $\theta^k_1(0,\ \tau)$ is assigned when the solution of equation (3) under condition (4) is found:

$$\Theta_l^{\kappa+1} = \Theta_1^{\kappa} + (1 - \Theta_1^{\kappa})\exp(-\beta x). \tag{11}$$

2. At discrete values of x the corresponding number of Stefan's problems is solved, where boundary condition (5) uses the liquid temperature value from formula (11).

3. The value of θ_1 thus found is used to define the liquid temperature more accurately. The procedure is repeated unless the condition

is satisfied, where n is the number of points on the x axis at which the liquid temperature is determined.

The computation experiment was carried out under the following values of the initial data: $\lambda_1 = 1.865$; $\lambda_2 = 2.874$; $\chi = 0.286$; the spatial grid step along the coordinate is 0.5 with the number of nodes equal to 100; the number of nodes along the x axis is 10.

We studied the effect of the conditions for heat exchange between the liquid flow and the ground, the flow velocity, and the ratio of inlet to initial ground temperature. Accordingly, the parameters α (10 and 20), β (0.1 and 1.0), θ_{ph} (0.091 and 0.286) were varied.

$$\max_n |\Theta_l^{\kappa+1} - \Theta_l^{\kappa}| < \varepsilon,$$

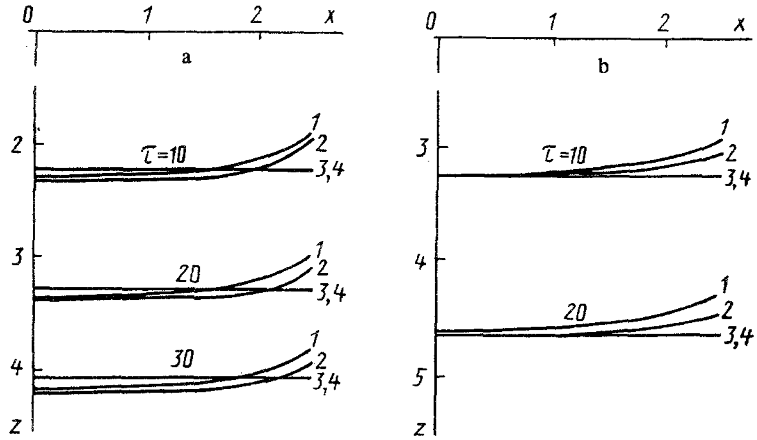

Fig. 1. Downstream variations of the thawed layer thickness (a, θ_{ph} = 0.286; b, 0.091): 1) β = 1.0; α = 10; 2) 1.0 and 20; 3) 0.1 and 10; 4) 0.1 and 20.

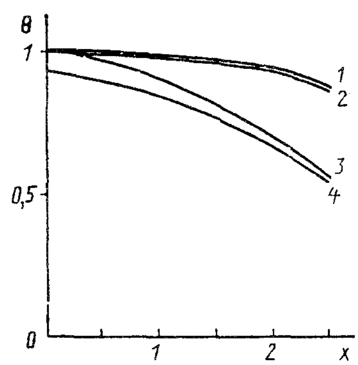

Fig. 2. Downstream variations of the liquid and ground temperature: 1, 3) liquid temperature at s = 5.0 and 0.5; 2, 4) ground temperature at z = 0 for the same values of s.

The computation results are plotted in Figs. 1 and 2. From Fig. 1 it follows that the ground thawing magnitude is most significantly affected by the water velocity and by the initial water-to-ground temperature (parameters β and θ_{ph}) ratio. The effect of the heat exchange conditions is less significant and is very inconsiderable (curves 3 and 4, in Fig. 1) at large water velocities.

Figure 2 plots the computation results on the liquid and ground surface temperatures at small liquid velocities (β = 1), small temperature head (θ_{ph} = 0.286) and at α = 10. It is seen that at small times these temperatures significantly differ (curves 3, 4); however, then they are practically flattened (curves 1, 2). This allows the initial problem to be simplified by substituting a first-kind boundary condition for condition (5).

NOTATION

α, thermal diffusivity; λ, thermal conductivity; c, ρ, specific heat capacity and density, respectively; h, V, liquid depth and velocity; T, temperature; t, time; α, coefficient of heat exchange between the liquid and rock; $\bar{s}$, interface; ℓ, specific heat of the phase transition; ω, ground icing; L, flow length. Indices: 1, thawed zone; 2, frozen; l, liquid; in, inlet cross section; 0, initial state; ph, phase transition.

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